FREE RESOURCE · ECAT / BCAT MATHS

# Maths Formula Sheet

Key formulas from the HSSC (FSc) Mathematics course, grouped by topic. Useful for NUST, FAST, GIKI, PIEAS and other engineering tests.

## Algebra

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$a^2 - b^2 = (a - b)(a + b)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ — quadratic formula

$\Delta = b^2 - 4ac$ — discriminant

$\alpha + \beta = -b/a,\ \alpha\beta = c/a$ — sum, product of roots

## Logarithms

$\log(mn) = \log m + \log n$

$\log(m/n) = \log m - \log n$

$\log m^n = n \log m$

$\log_a b = \log b / \log a$ — change of base

$\log_a a = 1,\ \log_a 1 = 0$

## Sequences and series

$a_n = a + (n - 1)d$ — A.P. term

$S_n = (n/2)[2a + (n - 1)d]$ — A.P. sum

$a_n = ar^{n-1}$ — G.P. term

$S_n = a(1 - r^n) / (1 - r)$ — G.P. sum, $r \neq 1$

$S_\infty = a / (1 - r)$ — $|r| < 1$

$A = (a + b)/2,\ G = \sqrt{(ab)}$ — A.M., G.M.

## Permutations, combinations, binomial

${}^nP_r = n! / (n - r)!$

${}^nC_r = n! / [r!(n - r)!]$

$(a + b)^n = \Sigma\ {}^nC_r\ a^{n-r}\ b^r$ — $r = 0 \ldots n$

$T_{r+1} = {}^nC_r\ a^{n-r}\ b^r$ — general term

## Trigonometric identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = (\tan A \pm \tan B) / (1 \mp \tan A \tan B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

$\tan 2A = 2\tan A / (1 - \tan^2 A)$

## Triangles

$a / \sin A = b / \sin B = c / \sin C$ — law of sines

$c^2 = a^2 + b^2 - 2ab \cos C$ — law of cosines

$\text{Area} = \tfrac{1}{2}\, ab \sin C$

## Coordinate geometry

| | |
|---|---|
| $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ | distance |
| $M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$ | midpoint |
| $m = (y_2 - y_1) / (x_2 - x_1)$ | slope |
| $y = mx + c$ | slope–intercept |
| $y - y_1 = m(x - x_1)$ | point–slope |
| $m_1 = m_2 \mid m_1 m_2 = -1$ | parallel \| perpendicular |
| $\lvert ax_1 + by_1 + c \rvert / \sqrt{(a^2 + b^2)}$ | point to line |
| $(x - h)^2 + (y - k)^2 = r^2$ | circle |
| $x^2 + y^2 + 2gx + 2fy + c = 0$ | centre $(-g, -f)$, $r = \sqrt{(g^2 + f^2 - c)}$ |
| $y^2 = 4ax$ | parabola, focus $(a, 0)$ |

## Vectors and matrices

| | |
|---|---|
| $a \cdot b = \lvert a \rvert \lvert b \rvert \cos\theta$ | dot product |
| $\lvert a \times b \rvert = \lvert a \rvert \lvert b \rvert \sin\theta$ | cross product |
| $\lvert a \rvert = \sqrt{(x^2 + y^2 + z^2)}$ | |
| $\det [a\ b; c\ d] = ad - bc$ | |
| $A^{-1} = (1/\det)[d\ -b; -c\ a]$ | 2 × 2 inverse |

## Limits and complex numbers

| | |
|---|---|
| $\lim (\sin x)/x = 1$ | $x \to 0$ |
| $\lim (1 + 1/n)^n = e$ | $n \to \infty$ |
| $i^2 = -1$ | |
| $\lvert a + bi \rvert = \sqrt{(a^2 + b^2)}$ | |

## Derivatives

| | |
|---|---|
| $d/dx\ (x^n) = nx^{n-1}$ | |
| $d/dx\ (\sin x) = \cos x$ | |
| $d/dx\ (\cos x) = -\sin x$ | |
| $d/dx\ (\tan x) = \sec^2 x$ | |
| $d/dx\ (e^x) = e^x$ | |
| $d/dx\ (\ln x) = 1/x$ | |
| $d/dx\ (a^x) = a^x \ln a$ | |
| $(uv)' = u'v + uv'$ | product rule |
| $(u/v)' = (u'v - uv') / v^2$ | quotient rule |
| $dy/dx = (dy/du)(du/dx)$ | chain rule |

## Integrals (+ C)

| | |
|---|---|
| $\int x^n\,dx = x^{n+1} / (n + 1)$ | $n \neq -1$ |
| $\int (1/x)\,dx = \ln\lvert x \rvert$ | |
| $\int e^x\,dx = e^x$ | |
| $\int \sin x\,dx = -\cos x$ | |
| $\int \cos x\,dx = \sin x$ | |
| $\int \sec^2 x\,dx = \tan x$ | |
| $\int a^x\,dx = a^x / \ln a$ | |
| $\int u\,dv = uv - \int v\,du$ | by parts |

## Standard values

| Θ | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| **sin** | 0 | 1/2 | 1/√2 | √3/2 | 1 |
| **cos** | 1 | √3/2 | 1/√2 | 1/2 | 0 |
| **tan** | 0 | 1/√3 | 1 | √3 | undefined |